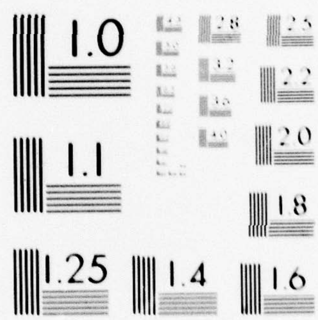
AD-A080 531 MASSACHUSETTS INST OF TECH LEXINGTON LINCOLN LAB F/G 9/5
SOME EFFECTS OF HARD LIMITING IN ADAPTIVE ANTENNA SYSTEMS.(U)
OCT 79 F W FLOYD, J T MAYHAN F19628-80-C-0002
UNCLASSIFIED TN-1979-15 ESD-TR-79-246 NL

1 OF 1
AD
A080531

END
DATE
FILMED
3-80
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

Technical Note	1979-15

F. W. Floyd, Jr.
J. T. Mayhan

Some Effects of Hard Limiting in Adaptive Antenna Systems

1 October 1979

Prepared for the Department of the Air Force
under Electronic Systems Division Contract F19628-80-C-0002 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

12

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

6

SOME EFFECTS OF HARD LIMITING IN ADAPTIVE ANTENNA SYSTEMS.

10 Franklin W. FLOYD, JR.
~~Group 68~~

Joseph T. MAYHAN
Group 61

14 TN-1979-15

9 TECHNICAL NOTE 1979-15

11 1 OCT[redacted]79

12 37

16 1227

18 ESD

15 F19628-80-C-0002

19 TR-79-246

Approved for public release; distribution unlimited.

LEXINGTON MASSACHUSETTS

207 650 [illegible]

ABSTRACT

Adaptive antennas are often implemented with the Applebaum-Howells type adaptive processor. Analog circuit versions of the Applebaum-Howells processor usually include a hard limiter between each antenna port and its correlation mixer, primarily for dynamic range compression. Brennan and Reed (IEEE Trans. Aeros. and Electron. Sys. AES-7, 68 [1971]) analyzed the effects of hard limiting, and their conclusions suggest that it does not degrade the steady-state performance of the adaptive processor.

This paper shows that hard-limited processors can perform much worse than those without hard limiting in situations where the correlation matrix of array signals has two or more eigenvalues of widely differing magnitudes and when a sensitivity threshold has been designed into the processor. The practical consequence is that (depending on the processor design parameters) when the adaptive antenna encounters two or more interference signals of different power levels, the larger signal can capture the hard limiter, allowing the smaller signals to pass through the processor unattenuated. Specific examples of this effect in both phased array and multiple beam adaptive antennas are presented.

CONTENTS

I. INTRODUCTION

Adaptive antennas have recently found wide application in radar and communications systems where it is necessary to cancel or "null" interference signals from sources which are spatially separated from the desired signal sources. A common implementation of an adaptive antenna is shown in Fig. 1a. It is an antenna array comprised of equal gain elements coupled with an Applebaum-Howells type adaptive processor (Refs. 1, 2). In this scheme the signals from the antenna elements are weighted and linearly combined to form a single output in which the interference signals from the various elements tend to cancel one another. The weight settings are determined by the processor feedback loops. A comprehensive analysis of the Applebaum-Howells technique can be found in the review paper by Gabriel (Ref. 3).

In analog circuit versions of the Applebaum-Howells processor, it has been common practice to insert a hard limiter between each antenna port and its correlation mixer as shown in Fig. 1b. There are two reasons for doing this. Both relate to the convenience of circuit implementation. At first, it would appear that the dynamic range required of the correlator and subsequent circuitry could be reduced considerably. (We will find that this is strictly true only in the special case of a single interference source.) Secondly, the correlators in adaptive antenna systems are often implemented with readily available, balanced diode mixers. This type of mixer is inherently a hard limiting device because one of its inputs, the local oscillator (LO) input, saturates the diodes and drives them in a switching mode, therefore the output is sensitive to the phase of this input, but not to its amplitude.

Gabriel (Ref. 3) and Brennan and Reed (Ref. 4) analyzed the effects of hard limiting on the performance of Applebaum-Howells processors. Their conclusions suggest that hard limiting has no harmful effect on the fundamental principle of operation of the adaptive processor because no information

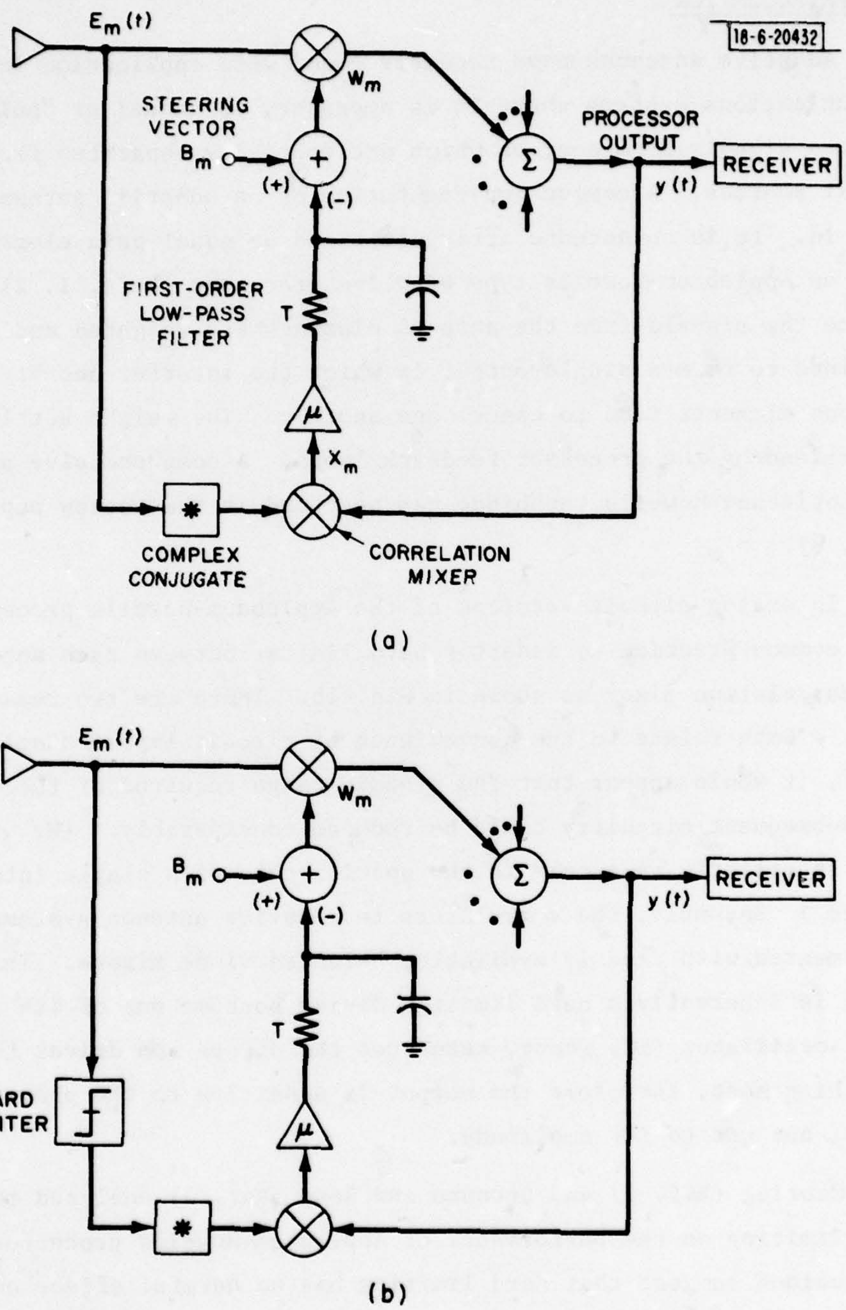


Fig. 1. Applebaum-Howells adaptive antenna processors: (a) standard, (b) hard-limited.

is lost by amplitude limiting one input of the correlator as long as the phase dependence is retained. This conclusion about hard limiters must be applied with caution. There are adaptive antenna applications for which the performance of a hard-limited processor is much worse than one without hard limiting. The problem with hard limiting arises when the correlation matrix of array signals has two or more eigenvalues with widely different magnitudes and when the processor has a sensitivity threshold. Multiple eigenvalues might arise, for example, from separate interference sources or from a single, broad-band source coupled with the frequency dispersion of the array, or from a combination of these effects. The sensitivity threshold of a processor is the power level below which it does not respond to interference signals. An example of a sensitivity threshold is found in the adaptive loop of Fig. 1a where the loop gain, μ, sets the threshold level. Threshold effects may also arise from preamplifier thermal noise, from noise in the post-correlator circuits, or from quantization levels in digital processors.

Figure 2 is a simple illustration of the problem. The computer-simulated performance of two Applebaum-Howells processors, with and without hard-limiting, is compared for the case of a linear array antenna of five, equal-gain elements. Both processors have the same sensitivity threshold. Two incident interference signals with power levels 10 dB and 40 dB above the sensitivity threshold are incident on the array from different directions. Since both signals are above threshold, the adaptive processor should null them both. The two adapted antenna gain patterns are shown in Fig. 2b along with the "quiescent" or unadapted pattern. The standard Applebaum-Howells processor (without the hard limiter) places nulls on both the large and small interference sources as desired whereas the hard limited processor nulls only the larger signal. The smaller signal, although far above threshold, is not attenuated at all. In this example, the hard-limited processor is totally ineffective in meeting its requirement, which is to cancel all interference signals above the threshold level.

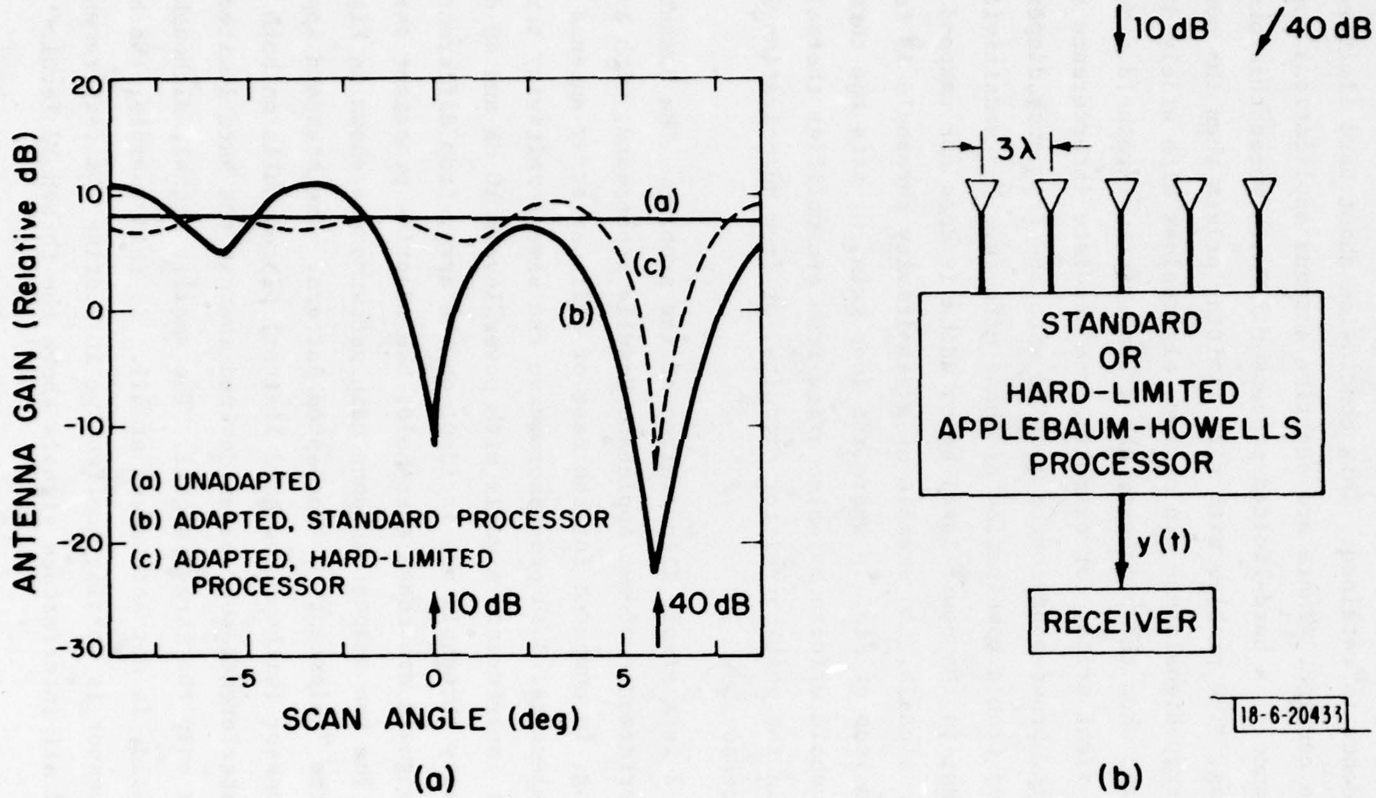


Fig. 2. Performance comparison of standard and hard-limited processors: (a) antenna array and processor, (b) radiation patterns.

In this paper we explore the reason for this failure and show that it is fundamental to the hard-limited correlation mixer. For phased array antennas, we present a simple analytical explanation based on Brennan and Reed's previous result (Ref. 4). Detailed computer simulations verifying the simple analysis are described. (It has also been verified in hardware.) We show that it is possible to circumvent this problem by increasing loop gain. However, in doing so, one gives up one of the major supposed advantages of hard limiting -- dynamic range compression. The use of hard limiting with multiple-beam antennas is discussed and compared with the phased array application. The hard-limiter effect on cancellation is not as severe in a multiple beam antenna as in a phased array; however, another problem arises -- that of pattern degradation.

We conclude that there is no theoretical advantage in hard limiting if more than one interference source (eigenvalue) is encountered, and there are some decided disadvantages. Practical considerations such as component availability may still dictate the use of hard limiting; however, its implications should be thoroughly understood before one proceeds.

II. PHASED ARRAYS

The sensitivity threshold, P_T, of an adaptive processor is often a basic design parameter. It can provide a simple means of discriminating between desired signals and interference signals on the basis of power level. It is often the case in radar and spread-spectrum communications applications that the minimum disruptive interference signal has greater power than the maximum desired signal. In these cases the sensitivity threshold can be set between these two power levels. It is then the task of the processor to sense all signals above threshold and null them to the extent that the total interference power at the processor output (receiver input) is equal to or less than P_T. In this way the receiver is guaranteed an acceptable interference level at its input and the desired signals are not sensed by the processor.

Figure 3 illustrates the threshold effect in standard and hard limited processors. It shows processor output power as a function of the input power from a single interference source for three cases: (a) unadapted, (b) adapted for a standard processor, and (c) adapted for a hard limited processor. The expressions for these curves are derived in Section III, "Analysis" (Eqs. (18) and (20)). The contribution of preamplifier thermal noise to the output power has been neglected as it does not contribute to an understanding of the effect we are discussing. For the purposes of this paper, we define the interference cancellation in decibels provided by the processor to be the difference between the unadapted and adapted curves. We will also assume that the maximum tolerable interference power at the processor output is 0 dB (on a relative scale) and that the largest interference power we are designing for is +40 dB. In other words, the cancellation requirement is 40 dB.

In comparing the two processors, note that no cancellation takes place in either processor when the input power, P_1, is less than the threshold power, P_T. As mentioned above, this fact is used to avoid the nulling of desired signals. At $P_1 = P_T$ the cancellation of both processors is 6 dB. As P_1 increases beyond P_T, the standard processor's output power decreases in inverse proportion to P_1 whereas the hard-limited processor's output reaches a level of P_T and remains constant at that level for larger values of P_1.

When P_1 is above threshold, the standard processor obviously provides twice the cancellation in decibels of the hard-limited processor. However, since P_T is presumably an acceptable level of interference, the additional cancellation of the standard processor is not needed. Moreover, the hard-limited processor seems to require only half the output dynamic range in decibels of the standard processor, therefore we might expect it to be much easier to implement.

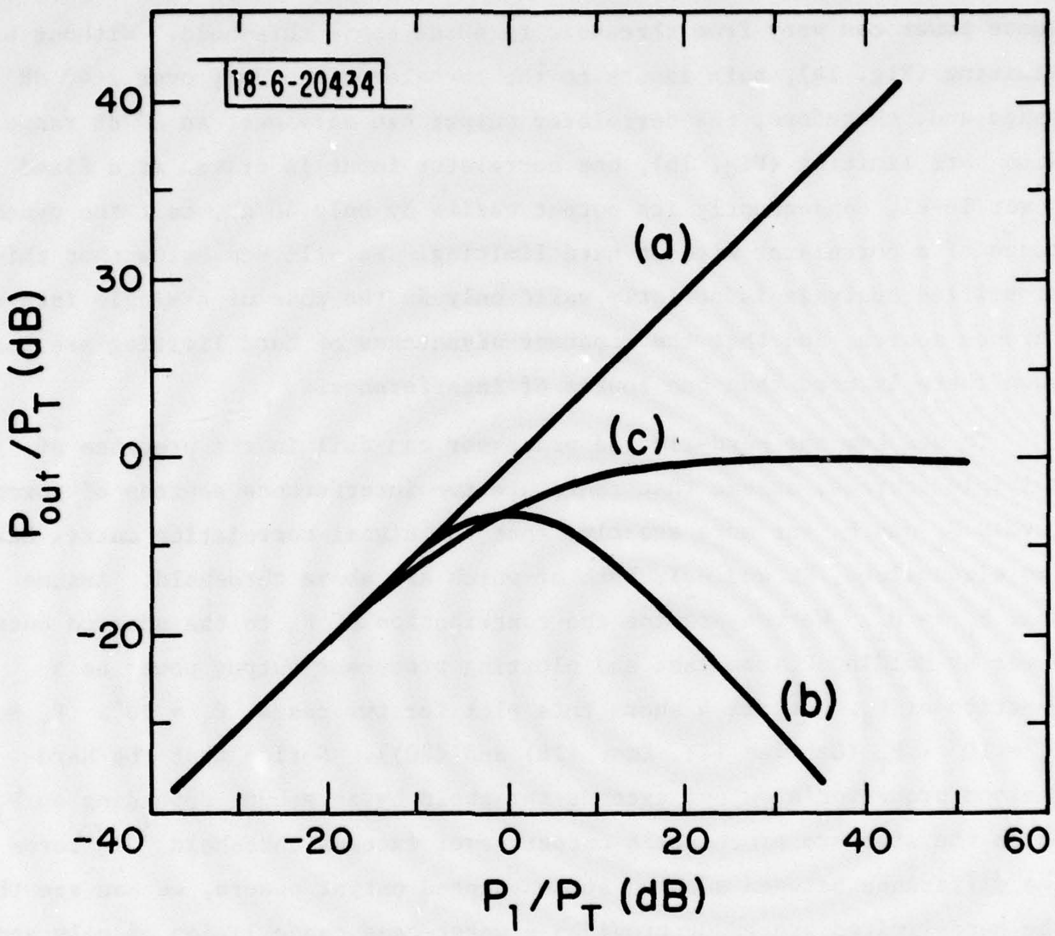


Fig. 3. Processor adapted output power vs input power, P_1, for a single interference source: (a) unadapted, (b) adapted, standard processor, (c) adapted, hard-limited processor.

To pursue this point further, consider a single interference source whose power can vary from threshold to 40 dB above threshold. Without hard limiting (Fig. 1a), both inputs to the correlator can vary over a 40 dB range and, therefore, the correlator output can vary over an 80 dB range. With hard limiting (Fig. 1b), one correlator input is driven at a fixed power level, consequently its output varies by only 40 dB, half the dynamic range of a correlator without hard limiting. We will see below that this simplified analysis is strictly valid only in the case of a single interference source, and that the apparent advantages of hard limiting are lost when there is more than one source of interference.

To see how the hard-limited processor can fail in the presence of multiple sources, assume that there are two interference sources of power levels P_1 and P_2 (or more exactly, that the signal correlation matrix has two eigenvalues, P_1 and P_2), both of which are above threshold. Assume that $P_1 >> P_2$. We can examine the contribution of P_2 to the adapted output power by holding P_1 constant and plotting processor output power as a function of P_2. Figure 4 shows this plot for two cases, $P_1 = 10^2 \cdot P_T$ and $P_1 = 10^4 \cdot P_T$ (Section III, Eqs. (18) and (20)). Notice that the hard-limited processor's output exceeds threshold by an amount depending on P_1 while the standard processor's output never exceeds threshold. In terms of the difference between adapted and unadapted output powers, we can see that the hard-limited processor provides a worst-case cancellation of only about 26 dB instead of the required 40 dB. This behavior of the hard-limited processor can be interpreted in terms of a variable sensitivity threshold for the smaller interference signal. We will see in Section III (Eq. (20)) that the smaller signal is not attenuated until it exceeds a value of $P_2 = \sqrt{P_1 P_T}$. This behavior is also evident from Fig. 4. We can interpret $\sqrt{P_1 P_T}$ as being an effective sensitivity threshold for the smaller signal, the level of which is set by the larger signal. In the example of Fig. 2, for instance, the smaller interference source is essentially unattenuated because its power is only 10 dB above the desired threshold, P_T, whereas the effective sensitivity threshold for that particular case is $\sqrt{P_1 P_T} = 20$ dB.

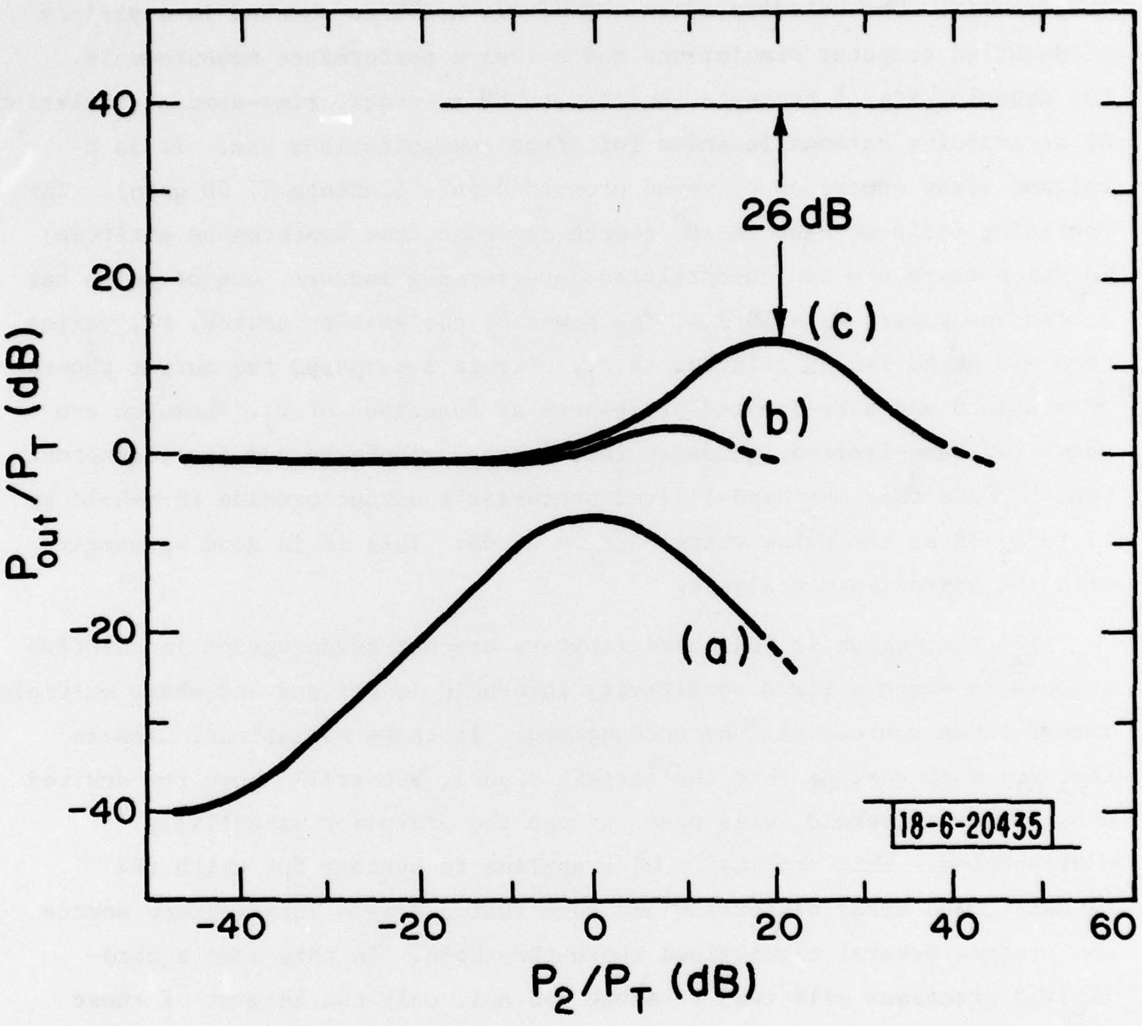


Fig. 4. Processor adapted output power vs power of second interference source, P_2: (a) standard processor, $P_1 = 10^4 \cdot P_T$, (b) hard-limited processor, $P_1 = 10^2 \cdot P_T$, (c) hard-limited processor, $P_1 = 10^4 \cdot P_T$.

These results are based on the simplified analysis presented in Section III. The validity of this analysis has been checked in a variety of detailed computer simulations and hardware performance measurements. For example, Fig. 5 presents the results of an exact, time-domain simulation of an adaptive antenna intended for space communications use. It is a thinned array comprised of seven crossed-dipole elements (7 dB gain). The operating field-of-view is 18^{o} (earth coverage from synchronous altitude) in which there are two uncorrelated interference sources, one of which has a constant power, $P_1 = 10^4 P_T$. The power of the smaller source, P_2, varies from -10 dB to +40 dB relative to P_T. Figure 5 compares the output powers of standard and hard-limited processors as functions of P_2. Results are shown for band-limited, gaussian interference waveforms and for incoherent tones. Note that the hard-limited processor's output exceeds threshold by 13 to 17 dB at the point where P_2/P_T = 20 dB. This is in good agreement with the approximate analysis.

The conclusion is that hard limiters are not advantageous in adaptive processors where a fixed sensitivity threshold is desired and where multiple interference sources will be encountered. In these situations, signals that are much smaller than the largest signal, but still above the desired sensitivity threshold, will pass through the processor essentially unattenuated. This conclusion also applies to systems for which the bandwidth and array dispersion are such that a single interference source can produce several eigenvalues above threshold. In this case a hard-limited processor will tend to sense and null only the largest of these eigenvalues.

By lowering the threshold, P_T, it is possible to design a hard-limited processor that will guarantee a tolerable level of output interference for any input power within the design range (0 dB to 40 dB in this case). Lowering the threshold, however, negates one of the major incentives for hard limiting because the circuit dynamic range required of the hard-limited processor will be essentially the same as it would be for a standard

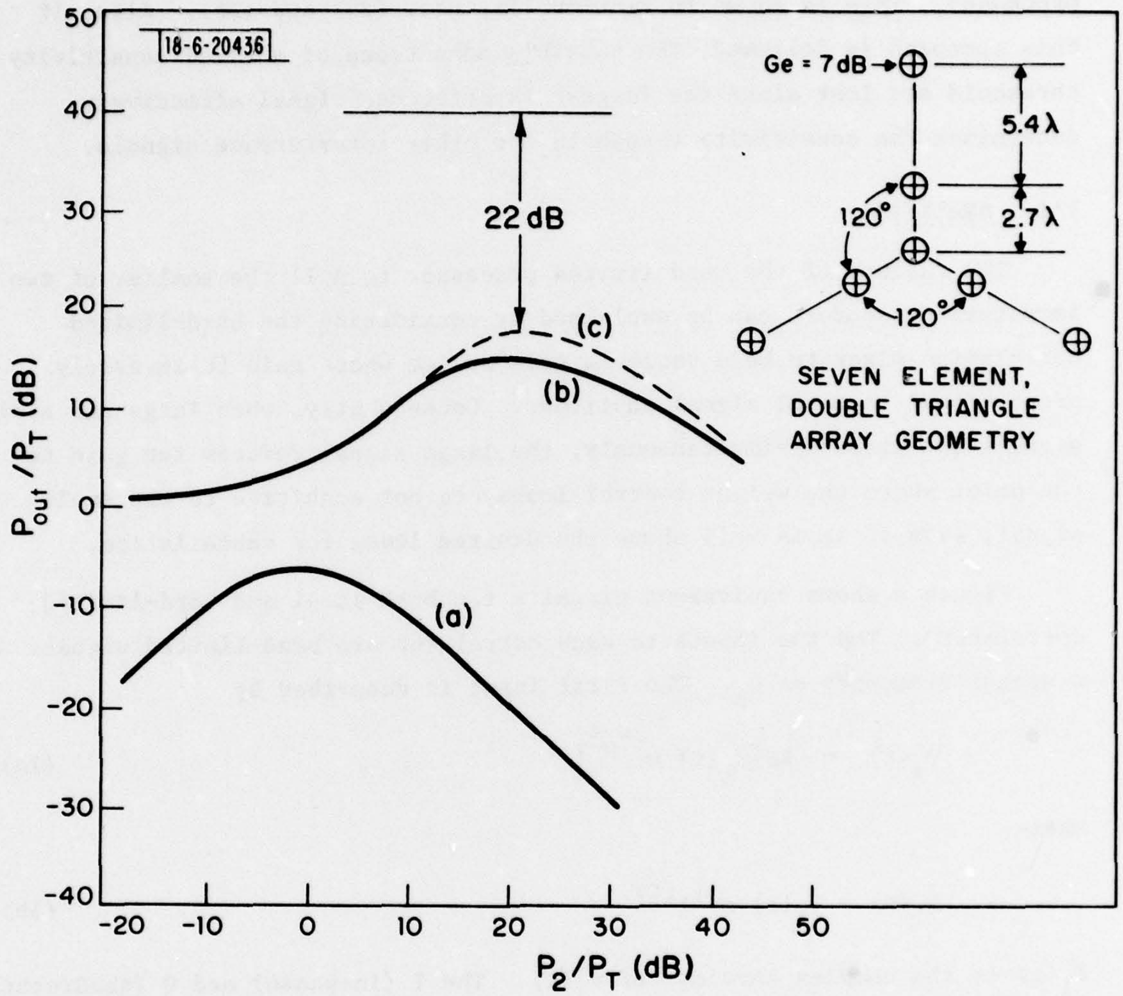


Fig. 5. Exact simulation result, processor adapted output power vs power of second interference source for antenna geometry shown, $P_1 = 10^4 \cdot P_T$, $\theta_1 = 0°$, $\phi_1 = 0°$, P_2 = variable, $\theta_2 = 7°$, $\phi_2 = 90°$: (a) standard processor, tone or gaussian interference, (b) hard-limited processor, gaussian interference, (c) hard-limited processor, tone interference.

processor. This is shown in Section III, Eqs. (25) and (26). Also, if this approach is followed, the possible advantages of a fixed sensitivity threshold are lost since the largest interference signal effectively determines the sensitivity threshold for other interference signals.

III. ANALYSIS

The failure of the hard limited processor to null the smaller of two interference sources can be explained by considering the hard-limited correlation mixer to be a variable gain device whose gain is inversely proportional to total signal amplitude. Consequently, when large and small signals are present simultaneously, the large signal reduces the gain to the point where the weight control loops are not sensitive to the small signal, even if it is well above the desired level for cancellation.

Figure 6 shows equivalent circuits for both ideal and hard-limited correlators. The two inputs to each correlator are band-limited signals at a center frequency of ω_o. The first input is described by

$$v_1(t) = \mathrm{Re}\left[E_1(t)\, e^{j\omega_o t}\right] \qquad (1a)$$

where

$$E_1(t) = V_1(t)\, e^{j\phi_1(t)} \qquad (1b)$$

$E_1(t)$ is the complex envelope of $v_1(t)$. The I (in-phase) and Q (quadrature) output voltages of the ideal correlator, after low-pass filtering, are given by

$$\begin{aligned} I(t) &= \frac{1}{4} V_1(t)\, V_2(t) \cos\left[\phi_2(t) - \phi_1(t)\right] \\ Q(t) &= \frac{1}{4} V_1(t)\, V_2(t) \sin\left[\phi_2(t) - \phi_1(t)\right] \end{aligned} \qquad (2)$$

Henceforth, the factor of $\frac{1}{4}$ will be included in the definition of loop gain, μ (Fig. 1). The units of μ are volts^{-2}. It is convenient to use

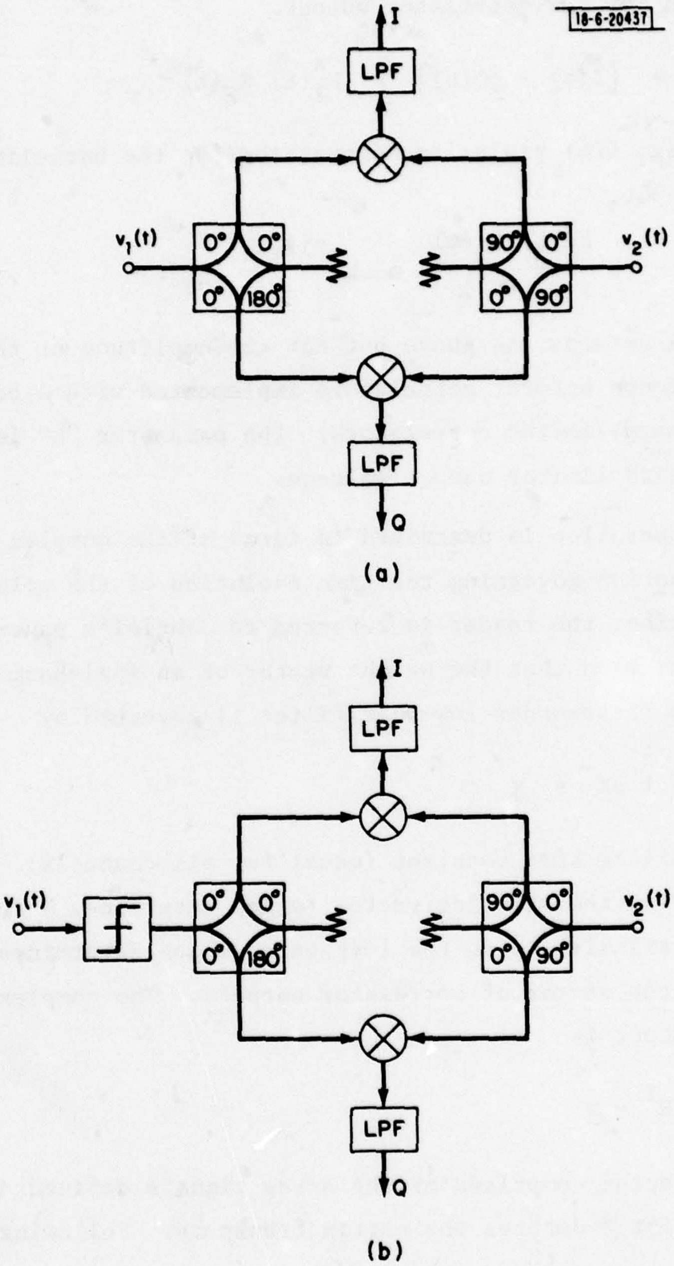


Fig. 6. Quadrature correlators: (a) standard, (b) hard-limited.

complex notation for the correlator output.

$$X(t) = \left[I(t) + jQ(t)\right] = E_1^*(t)\,E_2(t) \tag{3}$$

Inspection of Fig. 6(b) yields the expression for the hard-limited correlator's output.

$$X(t) = h\,\frac{E_1^*(t)\,E_2(t)}{|E_1(t)|} = he^{-j\phi_1(t)}\,E_2(t) \tag{4}$$

Notice that X(t) retains the phase but not the amplitude of the first input. As mentioned before, correlators implemented with diode mixers can be modelled as hard-limited correlators. The parameter "h" in Eq. (4) represents the hard limiter output voltage.

Processor operation is described in terms of the complex vector differential equation governing the time evolution of the weights. For a complete derivation, the reader is referred to Gabriel's paper (Ref. 3). From Fig. 1 it is seen that the weight vector of an Applebaum-Howells processor with a first-order low-pass filter is governed by

$$T\underline{\dot{W}} + \underline{W} + \mu\underline{X} = \underline{B} \tag{5}$$

where T is the filter time constant (equal for all channels), $\underline{W}$ is the weight vector, $\underline{B}$ is the steering vector (which determines $\underline{W}$ in the absence of interference signals), μ is the loop gain, which determines the sensitivity threshold. $\underline{X}$ is the vector of correlator outputs. The complex envelope of the processor output is

$$y = \underline{E}^T \cdot \underline{W} \tag{6}$$

where $\underline{E}$ is the vector comprised of the array signals defined as in Eq. (1) and the superscript T denotes the matrix transpose. Following Eqs. (3) and (4), the output of the n^{th} correlator is

$$\text{STANDARD} \qquad X_n = E_n^* \cdot y \tag{7a}$$

$$\text{HARD LIMITED} \qquad X_n = h \cdot \frac{E_n^*}{|E_n|} \cdot y \tag{7b}$$

We will make the usual assumption (Ref. 3) that the processor closed-loop bandwidth is small, therefore only the DC component of correlator output is significant in determining $\underline{W}$. Substituting Eq. (6) into (7), the DC components of the correlator outputs are

$$\text{STANDARD:} \qquad X_n = \sum_{m=1}^{N} \overline{(E_n^* E_m)} W_m \tag{8a}$$

$$\text{HARD LIMITED:} \qquad X_n = h \sum_{m=1}^{N} \overline{\left(\frac{E_n^* E_m}{|E_n|} \right)} W_m \tag{8b}$$

where the overbar denotes the DC component or expected value. Equation (8a) is easily expressed in terms of the correlation matrix of array signals, which is defined by

$$\underline{\underline{R}}_{nm} = \overline{E_n^* E_m} \tag{9}$$

Equation (8b) for the hard limiter cannot be simplified in general. In the special case of gaussian interference signals, Brennan and Reed found that the following simple relationship holds (Ref. 4).

$$\overline{\left(\frac{E_n^* E_m}{|E_n|} \right)} = \sqrt{\frac{\pi}{4 \overline{V_n^2}}} \; R_{nm} \tag{10}$$

where $\overline{V_n^2}$ is the mean squared magnitude of the n^{th} array element voltage. For an array of equal-gain elements $\overline{V_n^2} = \overline{V^2}$ for all n.

An expression for the steady-state adapted weight vector is obtained by setting $\dot{\underline{W}} = 0$ in Eq. (5) and using (8), (9) and (10).

$$\text{STANDARD:} \qquad \underline{W} = \left[\underline{\underline{I}} + \mu\underline{\underline{R}}\right]^{-1} \underline{B} \tag{11a}$$

$$\text{HARD LIMITED:} \qquad \underline{W} = \left[\underline{\underline{I}} + \mu h \sqrt{\frac{\pi}{4\,\overline{V^2}}}\;\underline{\underline{R}}\right]^{-1} \underline{B} \tag{11b}$$

In Eq. (11b) we have assumed equal interference power in all array elements, which restricts this analysis to arrays of identical antenna elements. Section IV discusses the case of Multiple-Beam Antennas in which the element powers are not equal.

Proceeding with the solution of Eq. (11a) for the standard processor, it is customary to work in the $\underline{\underline{R}}$ matrix eigenvector coordinate system (Ref. 3) where the solution for the adapted weights can be expressed as

$$\hat{W}_i = \frac{\hat{B}_i}{(1 + \mu P_i)} \quad ; \quad i = 1, 2, \ldots, N \tag{12}$$

where N is the number of antenna array elements. If $\underline{e}_i$ is the i^{th} eigenvector of $\underline{\underline{R}}$, then $\hat{W}_i = (\underline{e}_i^{\dagger} \cdot \underline{W})$ and $\hat{B}_i = (\underline{e}_i^{\dagger} \cdot \underline{B})$. P_i is the i^{th} eigenvalue of $\underline{\underline{R}}$.

For the hard-limited processor, the solution for the adapted weights (Eq. (11b)) is

$$\hat{W}_i = \frac{|\hat{B}_i|}{\left(1 + \mu h \sqrt{\frac{\pi}{4\overline{V^2}}}\; P_i\right)} \tag{13}$$

To relate $\overline{V^2}$ to the eigenvalues of $\underline{\underline{R}}$, it is well known that the sum of the diagonal elements (trace) of a matrix is equal to the sum of its eigenvalues.

$$\sum_{i=1}^{N} R_{ii} = \sum_{i=1}^{N} P_i \tag{14}$$

For the case of equal element power, $R_{ii} = \overline{V^2}$. Under the assumption of a single dominant eigenvalue, $P_1 >> P_2, P_3, \ldots, P_N$, the following approximation is very accurate:

$$N \overline{V^2} \approx P_1 \tag{15}$$

Equation (15) becomes

$$\hat{W}_i = \frac{|\hat{B}_i|}{\left(1 + \mu h \sqrt{\frac{\pi N}{4P_1}} P_i\right)} \tag{16}$$

The adapted output power of either processor can be calculated from

$$P_{out} = \sum_{i=1}^{N} |\hat{W}_i|^2 P_i \tag{17}$$

where the $\hat{W}_i$ are given by Eqs. (12) or (16). To compare the performance of the two processors, consider a simple case where there are only two non-zero eigenvalues of the correlation matrix. First, recall that the sensitivity threshold of a standard processor is $P_T = \mu^{-1}$. By substituting Eq. (12) into (13) we find an expression for the adapted output power of the standard processor:

$$\text{STANDARD:} \qquad P_{out} = |\hat{B}_1|^2 \frac{P_1}{\left(1 + \frac{P_1}{P_T}\right)^2} + |\hat{B}_2|^2 \frac{P_2}{\left(1 + \frac{P_2}{P_T}\right)^2} \tag{18}$$

We will assume that $P_1 >> P_2$. The corresponding definition for the sensitivity threshold of a hard-limited processor is

$$P_T = \left(\frac{\mu^2 h^2 N}{4}\right)^{-1} \tag{19}$$

With this expression, the hard-limited processor's output power is given by:

$$\text{HARD LIMITED:} \quad P_{out} = |\hat{B}_1|^2 \frac{P_1}{\left(1 + \frac{P_1}{\sqrt{P_1 P_T}}\right)^2} + |\hat{B}_2|^2 \frac{P_2}{\left(1 + \frac{P_2}{\sqrt{P_1 P_T}}\right)^2} \tag{20}$$

In comparing with Eqs. (18) and (20), note that P_T in Eq. (18) is replaced by $\sqrt{P_1 P_T}$ in Eq. (20). Since P_T represents a constant sensitivity threshold in the standard processor, the behavior of the hard-limited processor can be interpreted in terms of a variable threshold, which is given by $\sqrt{P_1 P_T}$. A large value of P_1 can raise the sensitivity threshold to the extent that P_2 is not sensed or nulled. Equations (18) and (20) are plotted in Figs. 3 and 4 assuming $|\hat{B}_1|^2 = |\hat{B}_2|^2 = 1$. Figure 3 shows the contribution of P_1 alone. Figure 4 shows the contribution of P_2 for two cases, $P_1 = 10^2 \cdot P_T$ and $P_1 = 10^4 \cdot P_T$.

By differentiating Eqs. (18) and (20), we find the maximum output interference power for the two processors (still assuming $|\hat{B}_1|^2 = |\hat{B}_2|^2 = 1$).

$$\text{STANDARD:} \quad P_{out\,max} = \frac{1}{2} P_T \; ; \text{ for } P_1 = P_2 = P_T \tag{21}$$

$$\text{HARD LIMITED:} \quad P_{out\,max} \approx \frac{1}{4} \sqrt{P_1 P_T} \; ; \text{ for } P_1 >> P_T \text{ and } P_2 = \sqrt{P_1 P_T} \tag{22}$$

Equations (20) and (22), derived from Brennan and Reed's result for gaussian waveforms (Ref. 4), show that a constant sensitivity threshold is not achievable in a hard-limited processor. By lowering the hard-limited threshold, P_T, the output interference power can be kept below a specified value; however, this increases the required cancellation capability of the processor and therefore the circuit dynamic range requirements. If P_I represents the tolerable interference power level at the processor's output, we would select P_T according to Eqs. (21) and (22). Allowing a few dB for margin, we would select:

$$\text{STANDARD:} \qquad P_T \approx 2P_I \tag{23}$$

$$\text{HARD LIMITED:} \qquad \sqrt{P_{max} P_T} \approx 4P_I \tag{24}$$

$$P_T \approx 16P_I^2/P_{max}$$

where P_{max} is the maximum input interference power for which we are designing. The hard-limited processor threshold is lower by a factor of $8P_I/P_{max}$ than the standard processor threshold. Recall that cancellation is the ratio of unadapted to adapted output power. From Eqs. (18) and (20) we have

$$\text{STANDARD:} \qquad C = P_{max}^2/P_T^2 = \frac{1}{4}(P_{max}/P_I)^2 \tag{25}$$

$$\text{HARD LIMITED:} \quad C = P_{max}/P_T = \frac{1}{16}(P_{max}/P_I)^2 \tag{26}$$

The required cancellation is practically the same in either case, therefore the circuit dynamic range requirements are hardly less severe in a hard-limited processor than in a standard one.

The variable threshold property of a hard-limited processor results from the variable gain of the correlator. Expressions for the correlator output vector are obtained by combining Eqs. (8a) and (9) for the standard correlator and (8b), (10) and (15) for the hard limited one.

$$\text{STANDARD:} \quad \underline{X} = \underline{\underline{R}} \cdot \underline{W} \tag{27}$$

$$\text{HARD LIMITED:} \quad \underline{X} = h \sqrt{\frac{\pi N}{4P_1}} \; \underline{\underline{R}} \cdot \underline{W} \tag{28}$$

The factor involving $\sqrt{P_1}$ can be interpreted as a variable gain. These equations hold for gaussian waveforms. Hard-limited correlators behave very similarly for tone inputs as can be shown by a simple example involving two incoherent tones. In Eqs. (3) and (4), let the correlator inputs be the sum of two sinusoids separated by $\Delta\omega$ in frequency.

$$E_1(t) = E_2(t) = S_1 + S_2 e^{j\Delta\omega t} \tag{29}$$

where $|S_1| >> |S_2|$. Substituting (29) into (3) gives the output of a standard correlator

$$X = |S_1|^2 + |S_2|^2 + 2\text{Re}\left[S_1^* \, S_2 \, e^{j\Delta\omega t}\right] \tag{30}$$

Because of the low-pass filter in the feedback loop (Fig. 1a) only the DC component of (30) is significant to the steady-state operation of the nulling processor. The output of a hard-limited correlator is obtained by substituting Eq. (29) into (4)

$$X = h|S_1 + S_2 \, e^{j\Delta\omega t}| = \sqrt{S_1^2 + S_2^2 + 2S_1S_2 \, \cos\Delta\omega t} \tag{31}$$

A Taylor series expansion of Eq. (31) to second order in $Z = S_2/S_1$ yields

$$X = h|S_1|\left[1 + \frac{1}{4} Z^2 + Z \cos\omega t - \frac{1}{4} Z^2 \cos 2\omega t\right] \quad (32)$$

Again, only the DC component is significant. The standard and hard-limited correlators are therefore characterized by the following expressions.

$$\text{STANDARD:} \quad X = \left[|S_1|^2 + |S_2|^2\right] \quad (33)$$

$$\text{HARD LIMITED:} \quad X = \frac{h}{|S_1|}\left[|S_1|^2 + \frac{1}{4}|S_2|^2\right] \quad (34)$$

In comparing Eqs. (33) and (34), the factor of $\frac{1}{4}$ in Eq. (34) is unimportant (it would be unity for gaussian instead of sinusoidial waveforms). The crucial difference in the two correlators is that the hard-limited version has a variable coefficient or "gain" which is inversely proportional to the amplitude of the larger signal. This property of a hard-limited correlator results in the variable sensitivity threshold of the processor because the threshold level is determined directly by the effective loop gain. This example indicates that hard-limited processors should behave similarly (within a few dB) for tone or gaussian inputs. This is borne out by the simulation results presented in Fig. 5.

IV. MULTIPLE-BEAM ANTENNAS

Most of the reported applications of the Applebaum-Howells adaptive processor have been in conjunction with phased array antennas; however, it can equally as well be used with multiple-beam antennas (Ref. 5). A multiple-beam antenna (MBA) illuminates the field of view (FOV) served by the antenna with a set of fixed-position beams (or radiation patterns), the composite of which covers the complete FOV. Generally, adjacent beams overlap at a gain level of about 4-5 dB down from the beam peak gain. For a satellite in geosynchronous orbit, earth coverage illumination is best

achieved (Ref. 6) by positioning the beams in a hexagonal pattern which results in a smaller center-to-center beam separation than a rectangular pattern. The total number of beams must be one of the series of numbers, 7, 19, 37, 61, ... in order to fully illuminate the circular FOV. To each beam set, there corresponds a given sized aperture consistent with this number of beams. As the number of beams within a fixed FOV is increased, the aperture size must increase correspondingly. For a large number of beams, there is significant coupling only between adjacent beams. Because of this, the correlation matrix defined at the beam output ports is diagonally dominant. Thus the loops of an adaptive feedback processor used in conjunction with an MBA tend to be partially decoupled. In fact, in the limit of an ideal set of narrow beams with no beam coupling, each loop of the processor can sense only a single interference source, and all the improvements in circuit dynamic range previously attributable to the hard limiter for a single source are realized. As a consequence of this loop decoupling, the spread in eigenvalues of the correlation matrix as sensed by a hard-limited processor can be considerably smaller when the processor is used in conjunction with an MBA than it would be for a phased array (Ref. 5). This eigenvalue compression often makes the smaller eigenvalues appear to be above the desired sensitivity threshold for an MBA in cases where they would be below threshold with an equivalent phased array.

The differences in performance for hard-limiting processors used with either phased arrays or MBA's can best be explained by generalizing Eq. (11b) so that it applies to MBA's as well. For the MBA, the mean squared output voltage, $|V_n|^2$, is different for each beam port. Define $P_n \equiv |V_n|^2$ and define the diagonal matrix $\underline{\underline{A}}$ according to

$$\underline{\underline{A}}_{n,m} = \sqrt{P_n}\, \delta_{n,m} \tag{35}$$

where $\delta_{n,m}$ is the Kronecker delta function: $\delta_{n,m} = 0$ for $n \neq m$; $\delta_{n,m} = 1$ for $n = m$. Modifying Eq. (10) in this way, substituting it into Eq. (5) and setting $\dot{\underline{W}} = 0$, we obtain for the steady-state adapted weight vector,

$$\underline{W} = [\underline{\underline{I}} + C\underline{\underline{A}}^{-1} \cdot \underline{\underline{R}}]^{-1} \cdot \underline{B} \tag{36}$$

where we have defined the constant $C = \mu h\sqrt{\pi/4}$. Equation (36) for the MBA can be compared to Eq. (11b) for the phased array. The most notable difference is that the adapted solution for $\underline{W}$ is now characterized by the eigenvalues and eigenvectors of $\underline{\underline{A}}^{-1} \cdot \underline{\underline{R}}$ for the hard-limited MBA, whereas it depends only on the eigenvalues and eigenvectors of $\underline{\underline{R}}$ for the hard-limited phased array. One can show that the effect of the matrix, $\underline{\underline{A}}^{-1}$ is to compress the spread in the eigenvalues of the matrix $\underline{\underline{A}}^{-1} \cdot \underline{\underline{R}}$ relative to those of $\underline{\underline{R}}$ alone. Quantitative estimates of the compression ratio as a function of the number of beams for a geosynchronous satellite, earth-coverage FOV are presented in Ref. (6). However, the eigenvectors and eigenvalues of $\underline{\underline{A}}^{-1} \cdot \underline{\underline{R}}$ are difficult to compare with those of $\underline{\underline{R}}$ on an analytic basis, so that a simple analysis of the characteristics of a hard-limiting MBA is not possible in the way that it is for a hard-limiting phased array. Instead, we present an intuitive development along with illustrative computer simulations of specific interference scenarios.

Consider now a hard-limited Applebaum-Howells processor used in conjunction with an MBA antenna configuration employing orthogonal beams. Orthogonal beams are ones for which the radiation pattern of all other beams have nulls at the beam maximum of any single port. With orthogonal beams, one anticipates good loop decoupling when one of the two sources is near a beam maximum, whereas maximum coupling occurs when the two sources are at adjacent triad locations (i.e., points where three beams intersect). In the case where two sources are positioned in distant beams, $\underline{\underline{R}}$ is nearly diagonal. $\underline{\underline{A}}^{-1} \cdot \underline{\underline{R}}$ is also nearly diagonal, therefore each loop acts nearly

independently of the others. Consequently, the most interesting situations occur when the two sources are in adjacent, coupled beams so that without loss in generality, we can restrict consideration to a 7-beam geometry.

To compare MBA performance with the phased array results of Fig. 5, we will again assume two interference sources, P_1 and P_2, set the power of $P_1 = 10^4 \cdot P_2$ and examine the quiescent and adapted output powers as P_2 is varied, using the 7-beam geometry shown in the insert of Fig. 7. Results are given in Fig. 7 for three different relative positionings of the two interference sources: adjacent beam peaks, adjacent triads, and a beam peak and adjacent triad. Observe that when either source is at a beam peak, both sources are sensed and nulled for all values of P_2/P_t. This is a consequence of the orthogonal beams and the resultant loop decoupling achieved when one source is at a beam maximum. When the sources are at adjacent triads, where maximum beam coupling occurs, there is a range of P_2/P_T over which the weaker signal is unattenuated. In this case the MBA behaves like the phased array and a worst-case cancellation of 29 dB results at a P_2 of approximately $\sqrt{P_1 P_t}$. This is 7 dB better than the 22 dB cancellation realized with the phased array under similar conditions (compare Fig. 7 with the tone results of Fig. 5). Figure 8 gives similar results for a 60-dB dynamic range system, where we set $P_1 = 10^6 \cdot P_T$. In this case a worse-case cancellation of 40 dB is achievable with the 60 dB dynamic range processor. This compares with a 34 dB cancellation for an equivalent hard-limited phased array processor. It appears, therefore, that hard-limited processors tend to give better cancellation performance when used with MBA's than with phased arrays.

It should be noted that the spatial resolution of the null formed on the interference sources is compromised somewhat by a hard-limited processor. To show this, we examine the solution for the adapted weights of an MBA processor with and without hard limiting. In the limit as loop gain approaches infinity we have

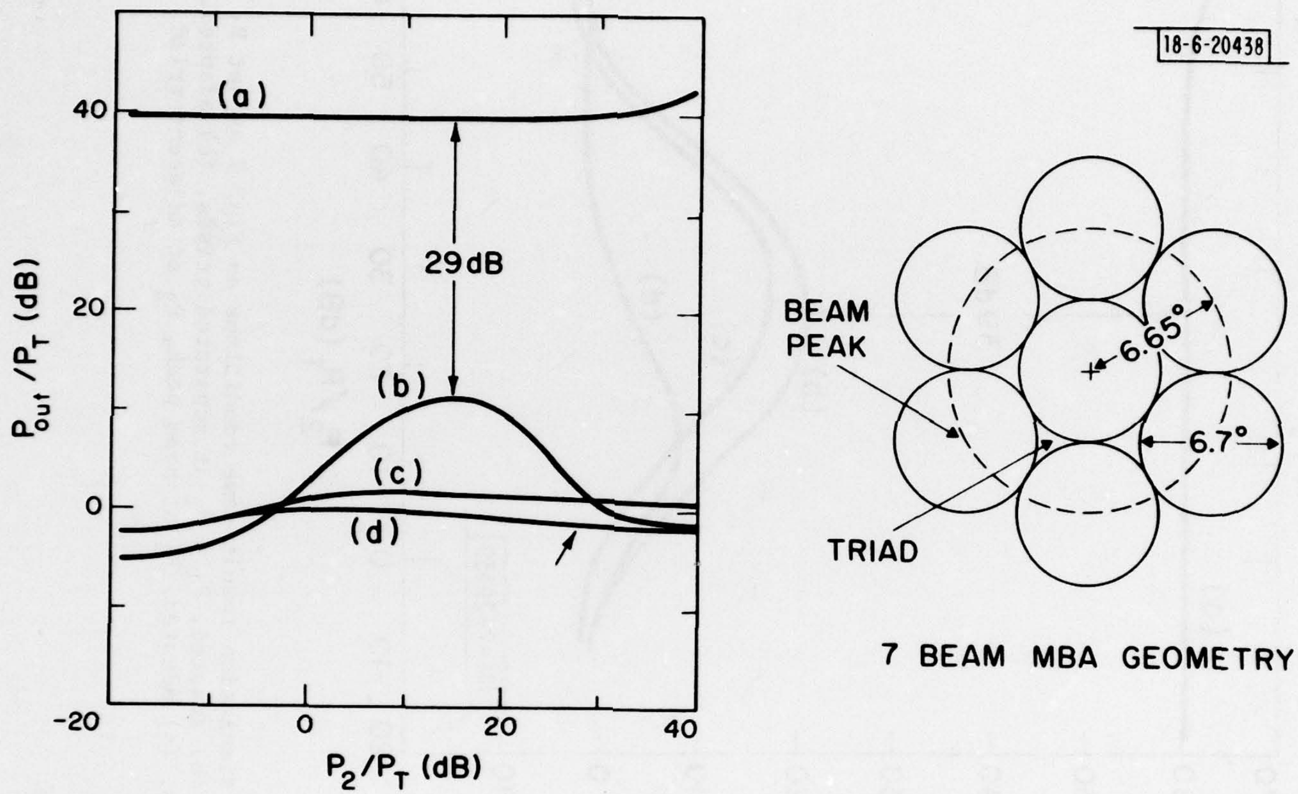


Fig. 7. 7 beam MBA geometry: exact simulation result, hard-limited processor output power vs power of second interference source, P_2, for MBA geometry shown, $P_1 = 10^4 \cdot P_T$, tone interference sources: (a) unadapted, (b) adapted, P_1 & P_2 at adjacent triads, (c) adapted, P_1 & P_2 at adjacent beam beaks, (d) adapted, P_1 at beam peak, P_2 at adjacent triad.

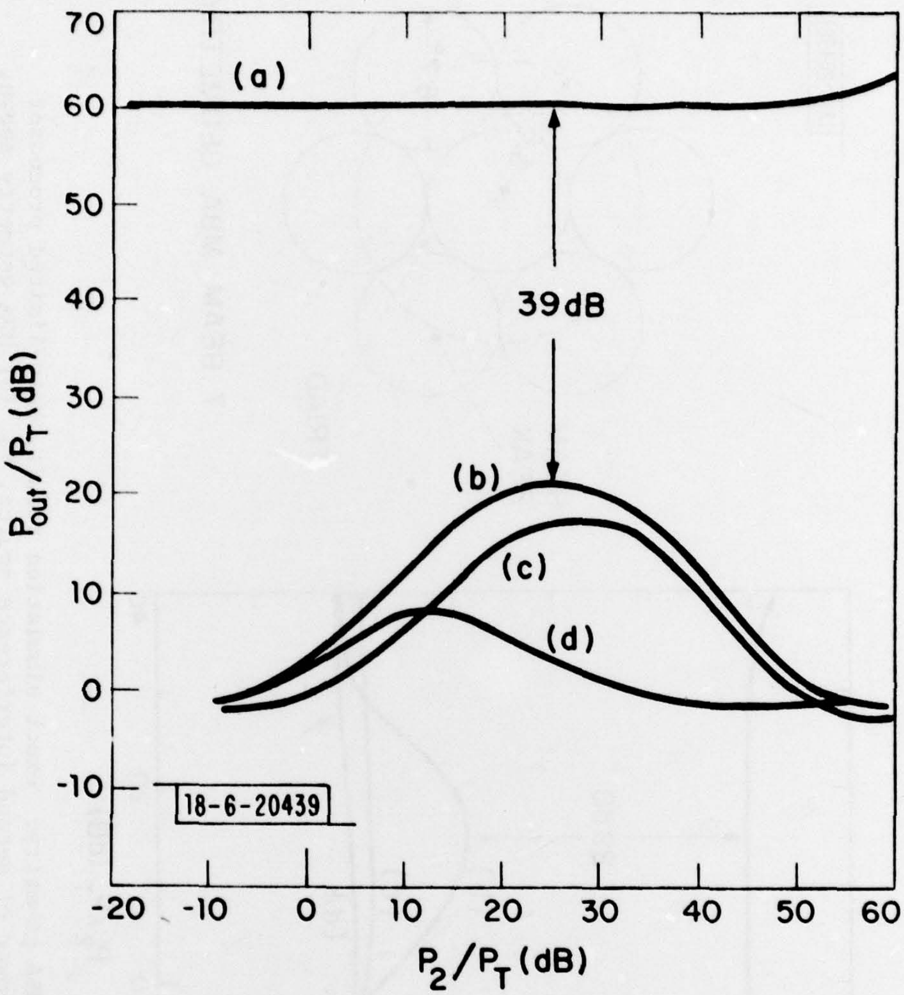


Fig. 8. Exact simulation result same conditions as Fig. 7 except $P_1 = 10^6 \cdot P_T$: (a) unadapted, (b) adapted, P_1 & P_2 at separated triads, (c) adapted, P_1 & P_2 at adjacent triads, (d) adapted, P_1 at beam peak, P_2 at adjacent triad.

$$\underline{W} = [\underline{\underline{I}} + C\underline{\underline{A}}^{-1} \cdot \underline{\underline{R}}]^{-1} \cdot \underline{B} \rightarrow \underline{\underline{R}}^{-1} \cdot (\underline{\underline{A}} \cdot \underline{B}) \tag{37}$$

Note that the effective steering vector is now $\underline{\underline{A}} \cdot \underline{B}$, and is dependent on the interference scenario. The beam which contains the largest source is weighted most heavily. We could compensate for this scenario-dependence measuring the output of each beam and applying a beam steering vector given by $\underline{\underline{A}}^{-1} \cdot \underline{B}_d$, where $\underline{B}_d$ is the desired steering vector. (In a phased array, $\underline{\underline{A}}$ is a constant times the identity matrix and the steering vector is inherently scenario-independent.) Figures 9 and 10 show typical radiation patterns of an MBA with and without a hard-limited processor corresponding to the scenario of Figure 7 with interference sources at adjacent beam peaks. Clearly the spatial resolution of the null in one plane has been compromised by the hard limiter. This effect would be disadvantageous in an application where the antenna is intended to serve a community of users spread uniformly over the FOV.

It should be mentioned in passing that there is perhaps a better way than hard limiting to achieve dynamic range compression in adaptive MBA systems. This is the scheme of weight pre-processing suggested in Ref. 6. The pre-weighting algorithm tends to equalize the power outputs from each of the antenna ports thus attenuating the larger interference sources to a level comparable with the smaller ones. In this case, the dynamic equation of the weights takes the form

$$T\dot{\underline{W}} + [\underline{\underline{I}} + \mu\underline{\underline{A}} \cdot \underline{\underline{R}}_o \cdot \underline{\underline{A}}] \cdot \underline{W} = \underline{B} \tag{38}$$

where $\underline{\underline{A}}$ is the diagonal matrix of attenuation factors applied by the pre-processor to each beam output port.

It is easy to see that $\underline{\underline{A}}$ does not affect the steering vector since it is diagonal. The expression

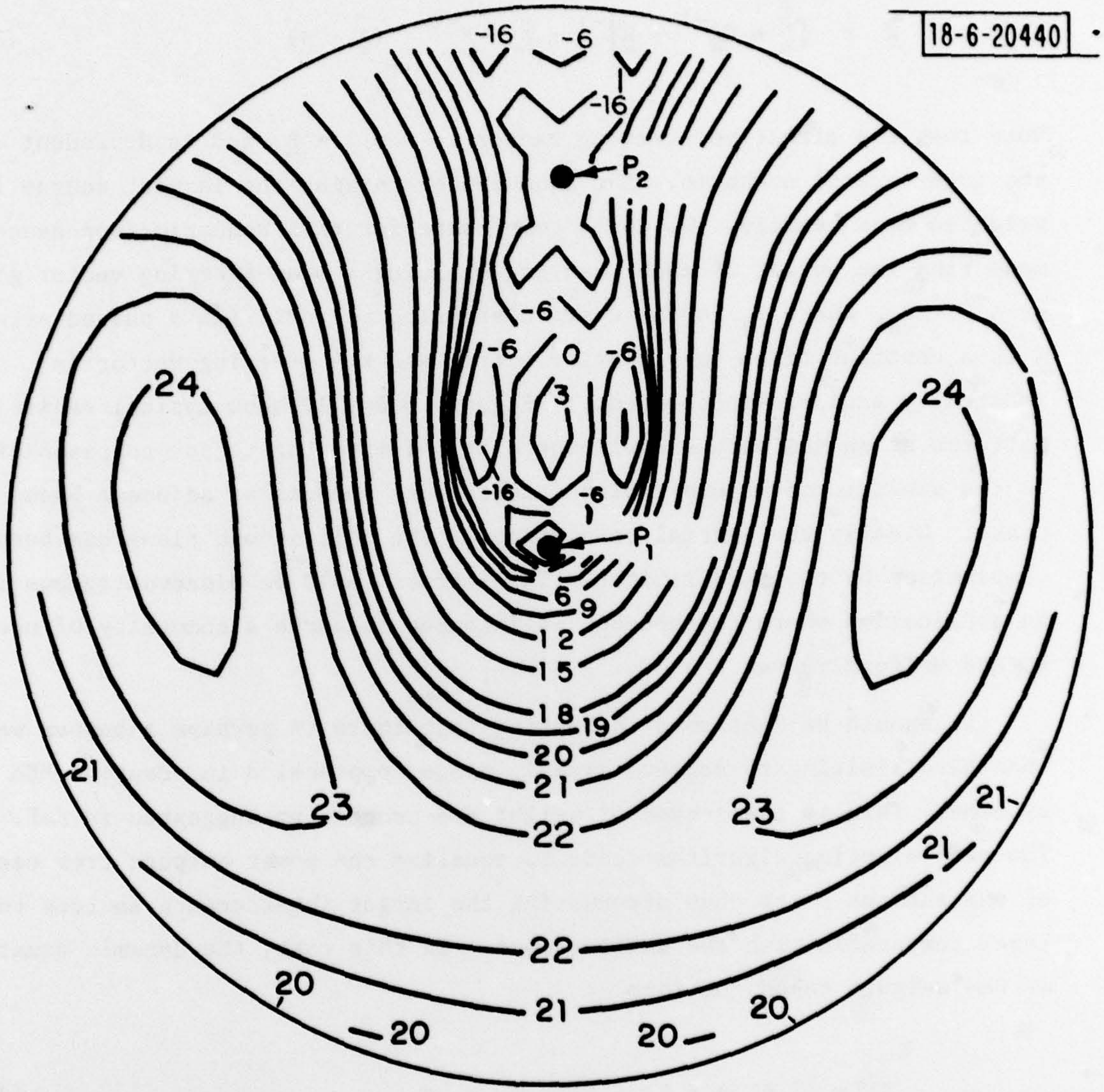


Fig. 9. Radiation pattern, standard processor, $P_1 = P_2 = 10^4 \cdot P_T$, P_1 & P_2 at adjacent beam peaks.

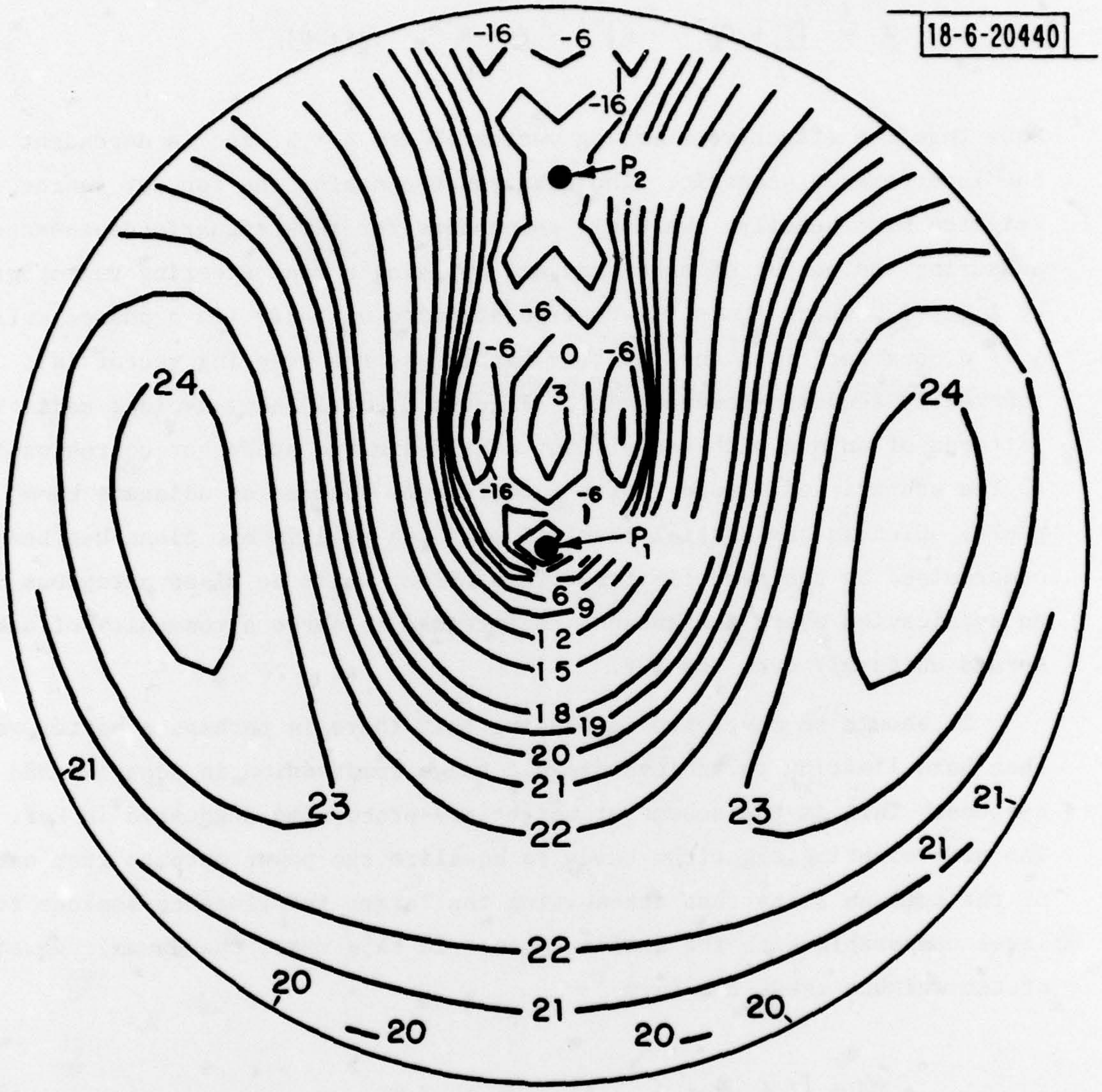


Fig. 9. Radiation pattern, standard processor, $P_1 = P_2 = 10^4 \cdot P_T$, P_1 & P_2 at adjacent beam peaks.

18-6-20441

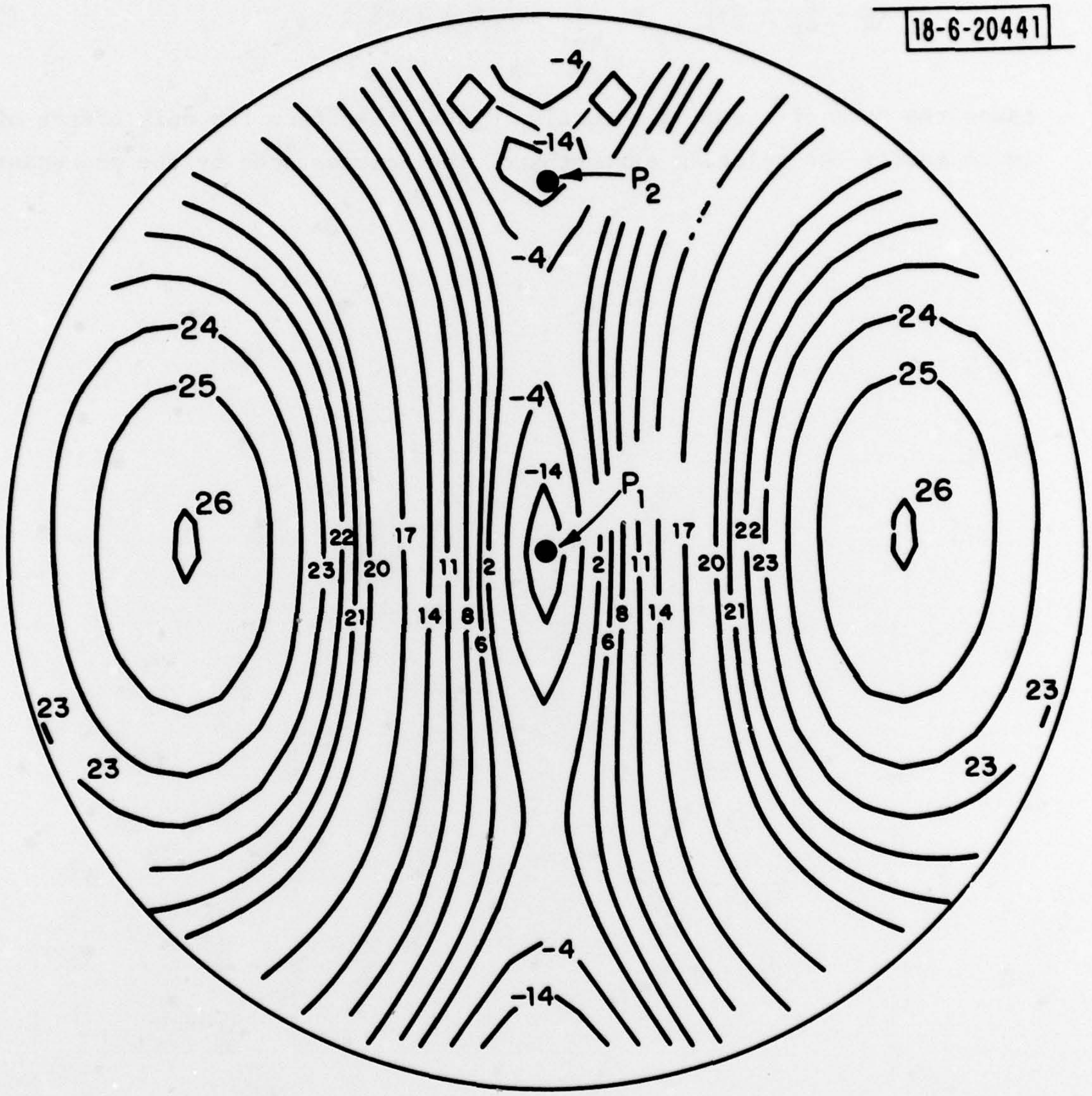


Fig. 10. Radiation pattern, same as Fig. 9 except hard-limited processor.

$$(\underline{\underline{A}} \cdot \underline{\underline{R}}_o \cdot \underline{\underline{A}})_{k,q} = \sum_{j=1}^{J} (A_k E_k)(A_q E_q) \tag{39}$$

takes the form of a sum of hermetian dyads, therefore the only effect of $\underline{\underline{A}}$ is to modify the relative strengths of the sources seen by the processor.

REFERENCES

1. P. W. Howells, "Intermediate Frequency Side-Lobe Canceller," U. S. Patent 3 202 990, August 24, 1965 (filed May 4, 1959).

2. S. P. Applebaum, "Adaptive Arrays," Syracuse University Research Corporation, Report SPL TR 66-1 (August, 1966).

3. W. F. Gabriel, "Adaptive Arrays - An Introduction," Proc. IEEE 64, 239 (1976).

4. L. E. Brennan and I. S. Reed, "Effect of Envelope Limiting in Adaptive Array Control Loops," IEEE Trans. Aerospace Electron. AES-7 , 698 (1971).

5. J. T. Mayhan, "Adaptive Nulling with Multiple-Beam Antennas," IEEE Trans. Antennas Propag. AP-26, 267 (1978).

6. A. R. Dion and L. J. Ricardi, "A Variable Coverage Satellite Antenna System," Proc. IEEE 59, 252 (1971).